Games

Part III: Dynamics — Episode 6

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“Dynamics” — Motivating a new “chapter”

- Sending packets through the Internet —
  - What are the design principles to make this happen?
  - How do we make it **fair** to all best-effort connections?
  - How do we support **performance guarantees** to those who need them?
- But intuitively, those who wish to have more “priorities,” “weights,” or “guarantees” need to, somehow, pay a price!
- But how?

This involves game-theoretic reasoning

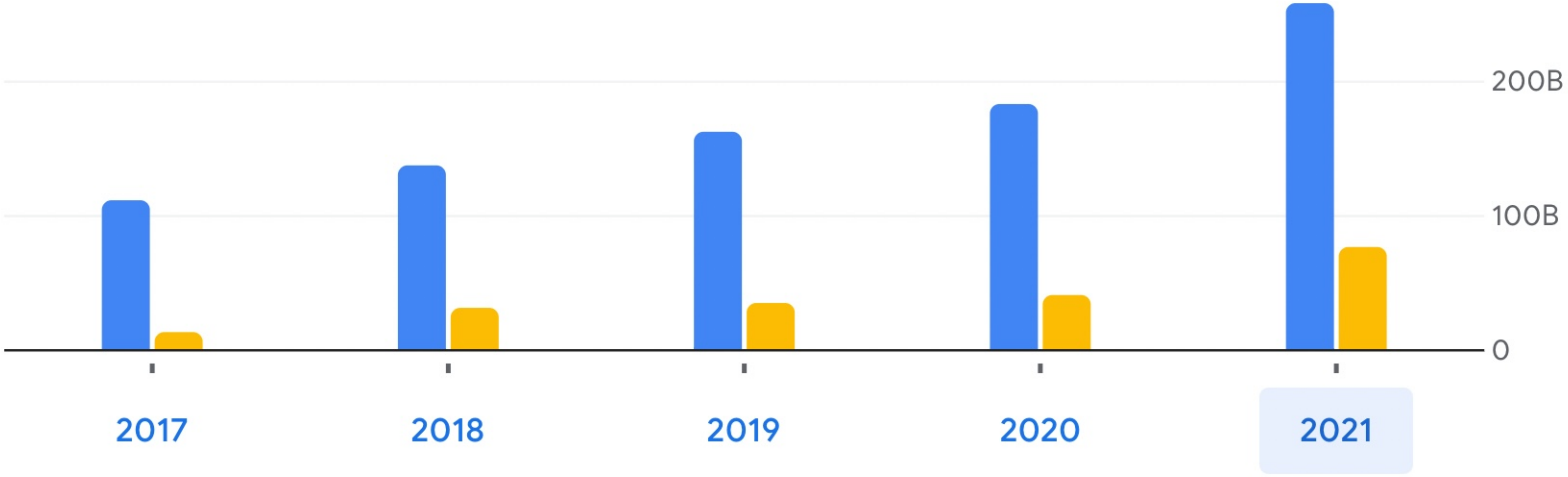
- All “peers” in a network make their individual decisions to maximize their own benefits
- To make it more general —
    - Rather than simply choosing a route in isolation, individual senders can evaluate routes in the presence of the congestion, resulting from the decisions made by themselves and everyone else
    - In Part III, we will develop models for network traffic using game-theoretic ideas
    - And show that adding capacity can sometimes **slow down** the traffic on a network

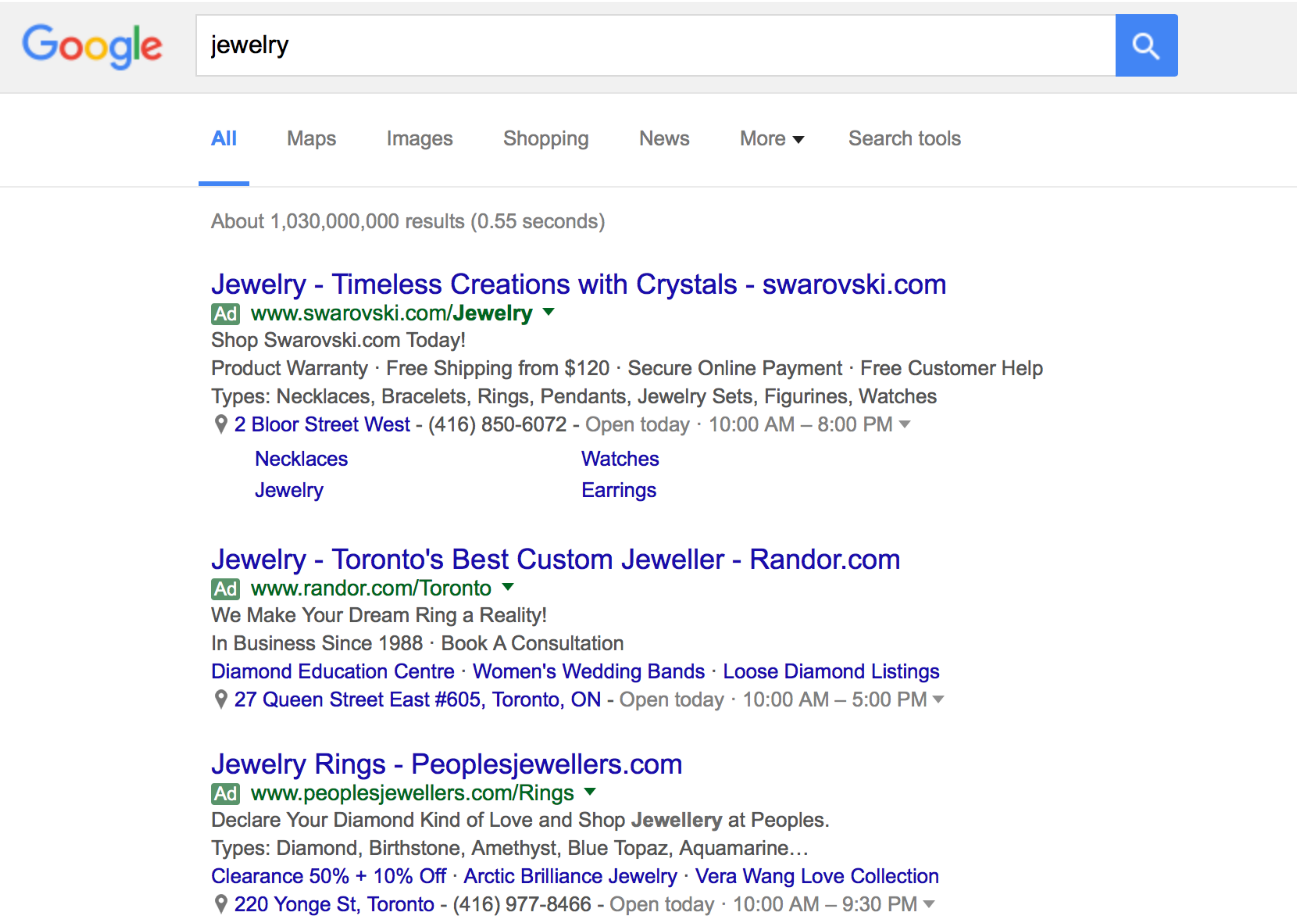
Viewing networks from a different perspective

- Traditionally, we view networks from the perspective of its underlying structure and architecture
- Now, we switch to look at an interdependence in the behaviour of the individuals who inhabit the system
  - The outcome for anyone depends on the combined behaviour of everyone
- Such interconnectedness at the level of behaviour can be studied using game theory

Another example of “dynamics” — **Google**

96% of the revenue ($64.4 billion in a quarter) is derived from advertising





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Adwords: keyword-based advertising

How does Google decide how much to charge for each ad?

To understand how ads are priced, we need to understand the fundamentals of auctions

To understand auctions, again, we need to understand the fundamentals of **games**

Textbook

Networks, Crowds, and Markets

(D. Easley and J. Kleinberg,
Cambridge University Press, July 2010)

Starting from Chapter 6

Freely downloadable from:
http://www.cs.cornell.edu/home/kleinber/networks-book/

Required reading: Chapter 6

What is a **game**? — A first example

- Suppose you are a college student
- Two pieces of work due tomorrow: an exam and a presentation
- You need to decide: study for the exam or prepare for the presentation?
  - Assuming you don’t have time to do both, and you can accurately estimate the grade

What is a game? — A first example

- Exam: **92** if you study, **80** if you don’t
- Presentation: You need to do it with a partner
  - If both of you prepare for it, both get **100**
  - If one of you prepares, both get **92**
  - If neither of you prepares, both get **84**

Basic ingredients of a game

- Participants in the game are called **players**
    - You and your partner
- Each player has a set of options for how to behave, referred to as the player's possible **strategies**
    - "Study for the exam" or "prepare for the presentation"
- For each choice of strategies, each player receives a **payoff**
    - The average grade you get on the exam and the presentation

A few assumptions to simplify the problem

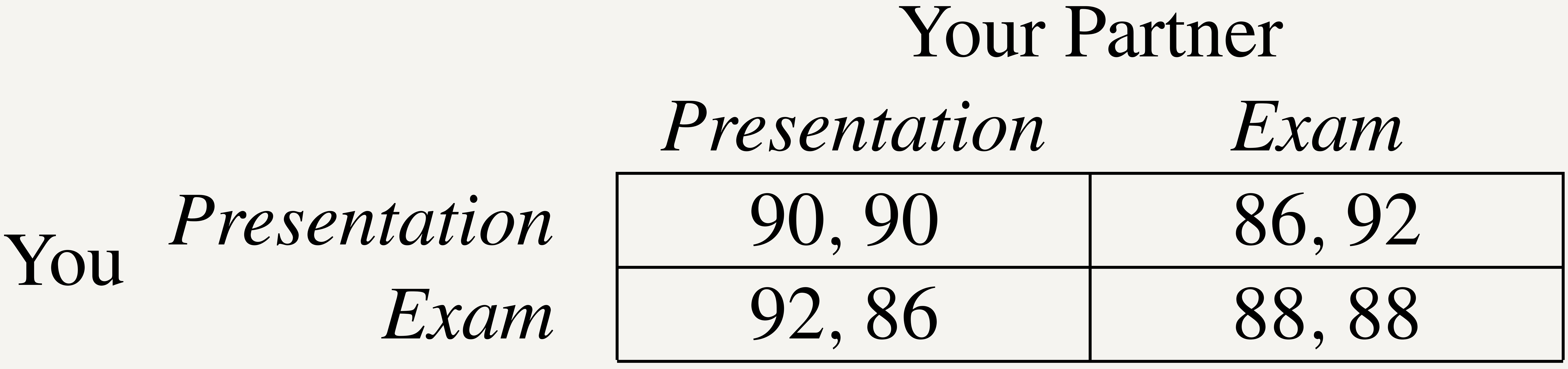
- Everything the player cares about is summarized in the player's payoffs
- Each player knows everything about the structure of the game
  - his own list of strategies
  - who the other player is
  - the strategies available to the other player
  - what her payoff will be for any choice of strategies

How do players select their strategies?

- Each player chooses a strategy to maximize her own payoff, given her beliefs about the strategy used by the other player — this is called **rationality**, and it implicitly includes two ideas:
    - each player wants to maximize payoff
    - each player actually succeeds in selecting the **optimal** strategy

Exam or presentation?

Exam or presentation?



		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1. Exam or Presentation?

Strictly dominant strategy

- A player has a strategy that is strictly better than all other options, regardless of what the other player does
  - In our example, studying for the exam is the strictly dominant strategy
  - A player will definitely play the strictly dominant strategy
  - This will be the outcome of the game

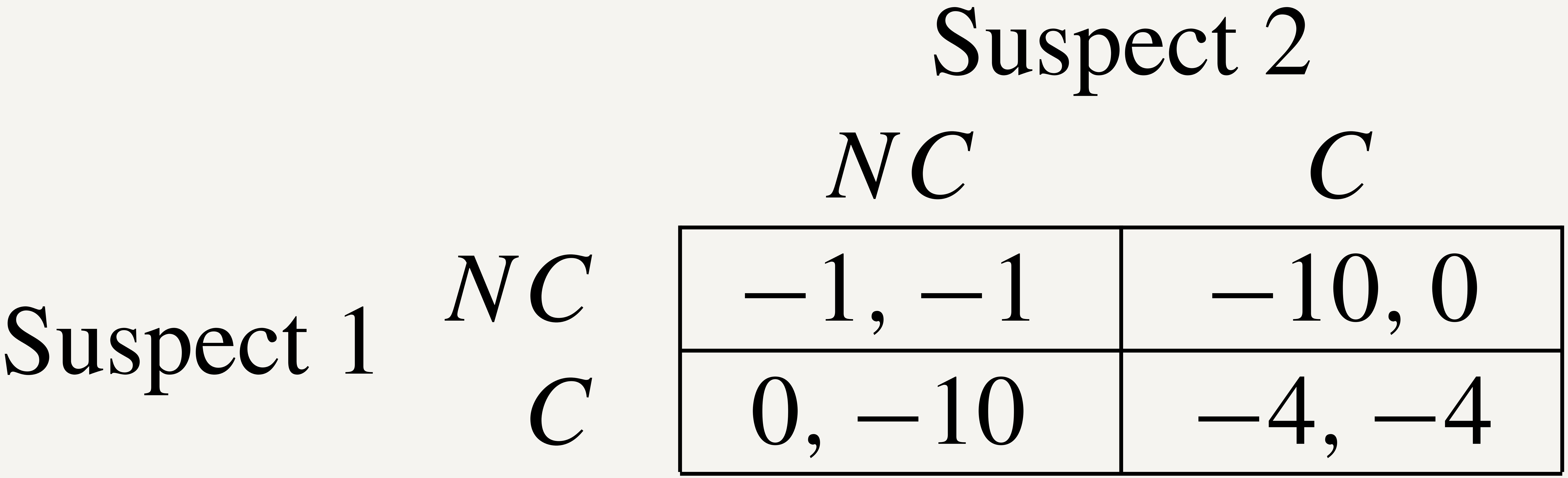
But there’s something striking

- If you and your partner could somehow agree that you would **both** prepare for the presentation, you will each get **90** as an average, and be better off

- But, despite that both of you understand this, the payoff of **90** cannot be achieved by rational play of this game! — why?

A related story: the Prisoner’s Dilemma

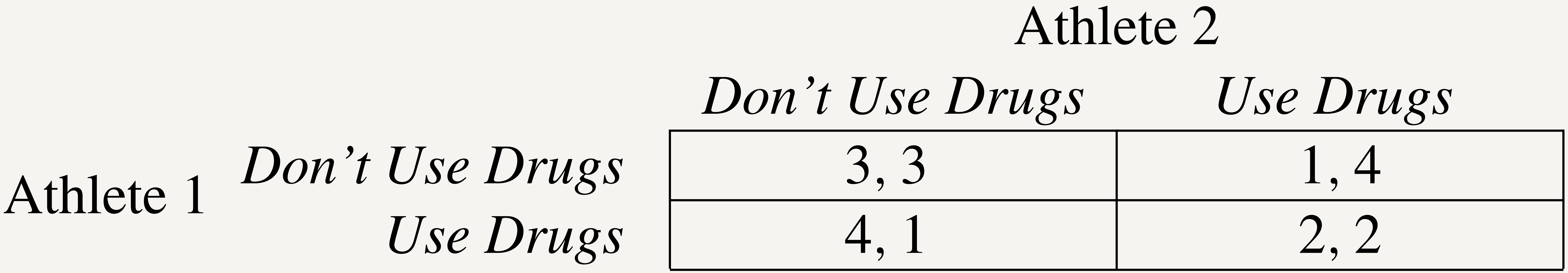
A related story: the Prisoner's Dilemma



		Suspect 2	
		NC	C
Suspect 1	NC	$-1, -1$	$-10, 0$
	C	$0, -10$	$-4, -4$

Figure 6.2. Prisoner's Dilemma

The “arms race” between competitors



		Athlete 2	
		Don’t Use Drugs	*Use Drugs*
Athlete 1	*Don’t Use Drugs*	3, 3	1, 4
	Use Drugs	4, 1	2, 2

Figure 6.3. Performance-Enhancing Drugs

Best responses

- If S is a strategy chosen by Player 1, and T is a strategy chosen by Player 2
- $P_1(S, T)$ denotes the payoff to Player 1 as a result of this pair of strategies (written in the payoff matrix in previous examples)
- A strategy S for Player 1 is a best response to a strategy T for Player 2, if S produces at least as good a payoff as any other strategy paired with T: $P_1(S, T) \geq P_1(S', T)$
- It is a strict best response if: $P_1(S, T) > P_1(S', T)$

Dominant strategies

- We say that a **dominant strategy** for Player 1 is a strategy that is a best response to **every** strategy of Player 2

- We say that a **strictly dominant strategy** for Player 1 is a strategy that is a **strict best response** to **every** strategy of Player 2

- In the Prisoner's Dilemma, both players had strictly dominant strategies

  - But this is not always the case!

The game of the marketing strategies

- People who prefer a low-priced version account for 60% of the population, and people who prefer an upscale version account for 40% of the population

- If a firm is the only one to produce a product for a given market segment, it gets all the sales

- Firm 1 is the much more popular brand, and so when the two firms directly compete in a market segment, Firm 1 gets 80% of the sales and Firm 2 gets 20% of the sales

		Firm 2	
		Low-Priced	*Upscale*
Firm 1	*Low-Priced*	.48, .12	.60, .40
	Upscale	.40, .60	.32, .08

Figure 6.5. Marketing Strategy

Only one player has a strictly dominant strategy

		Firm 2	
		Low-Priced	*Upscale*
Firm 1	*Low-Priced*	.48, .12	.60, .40
	Upscale	.40, .60	.32, .08

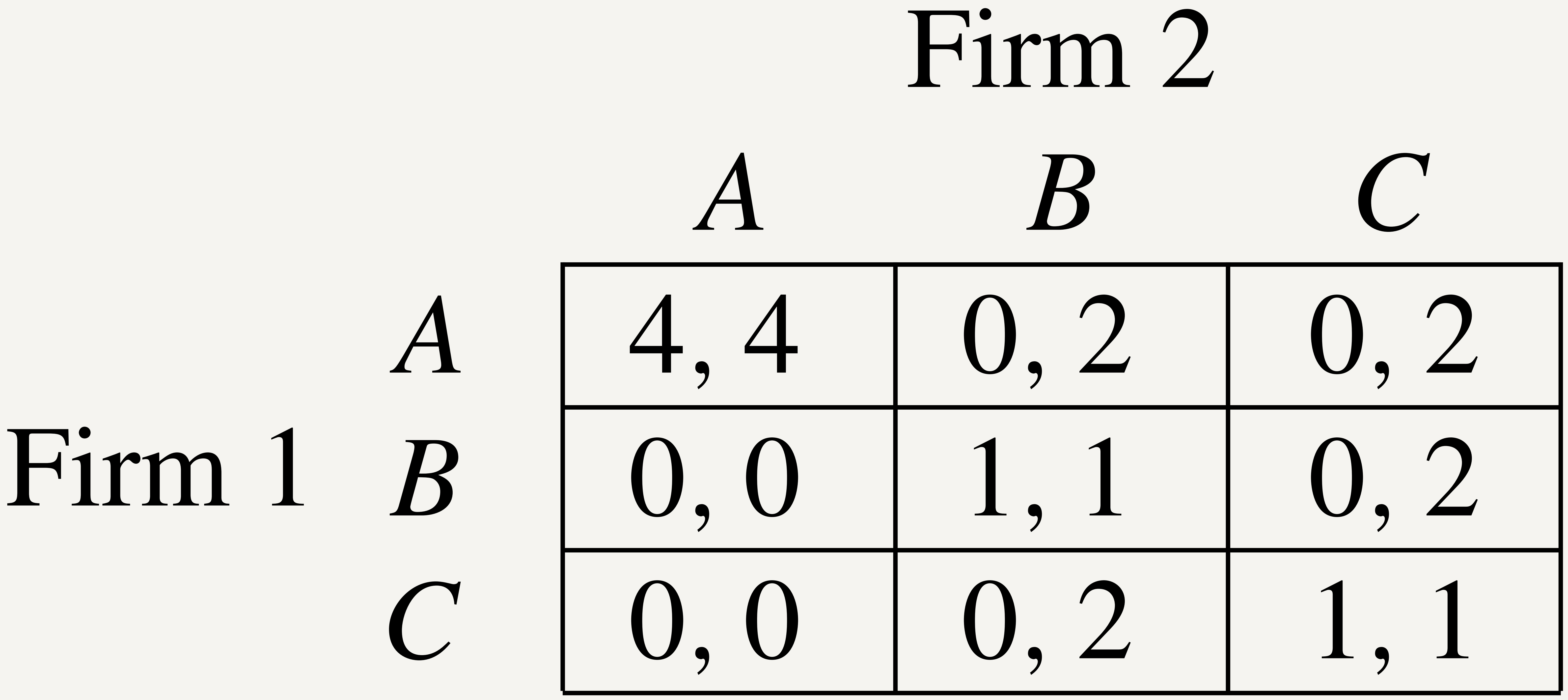
Figure 6.5. Marketing Strategy

Assumption: the players have **common knowledge** about the game: they know its structure, they know that each of them knows its structure, and so on

What if neither player has a strictly dominant strategy?

- Two firms and three clients: **A**, **B** and **C**
  - If the two firms approach the same client, the client will give half its business to each
  - **Firm 1** is too small to attract clients on its own, so if it approaches one client while **Firm 2** approaches a different one, then **Firm 1** gets a payoff of **0**
  - If **Firm 2** approaches client **B** or **C** on its own, it will get their full business. However, **A** is a larger client, and will only do business with both firms
  - Because **A** is a large client, doing business with it is worth **8**, whereas doing business with **B** or **C** is worth **2**

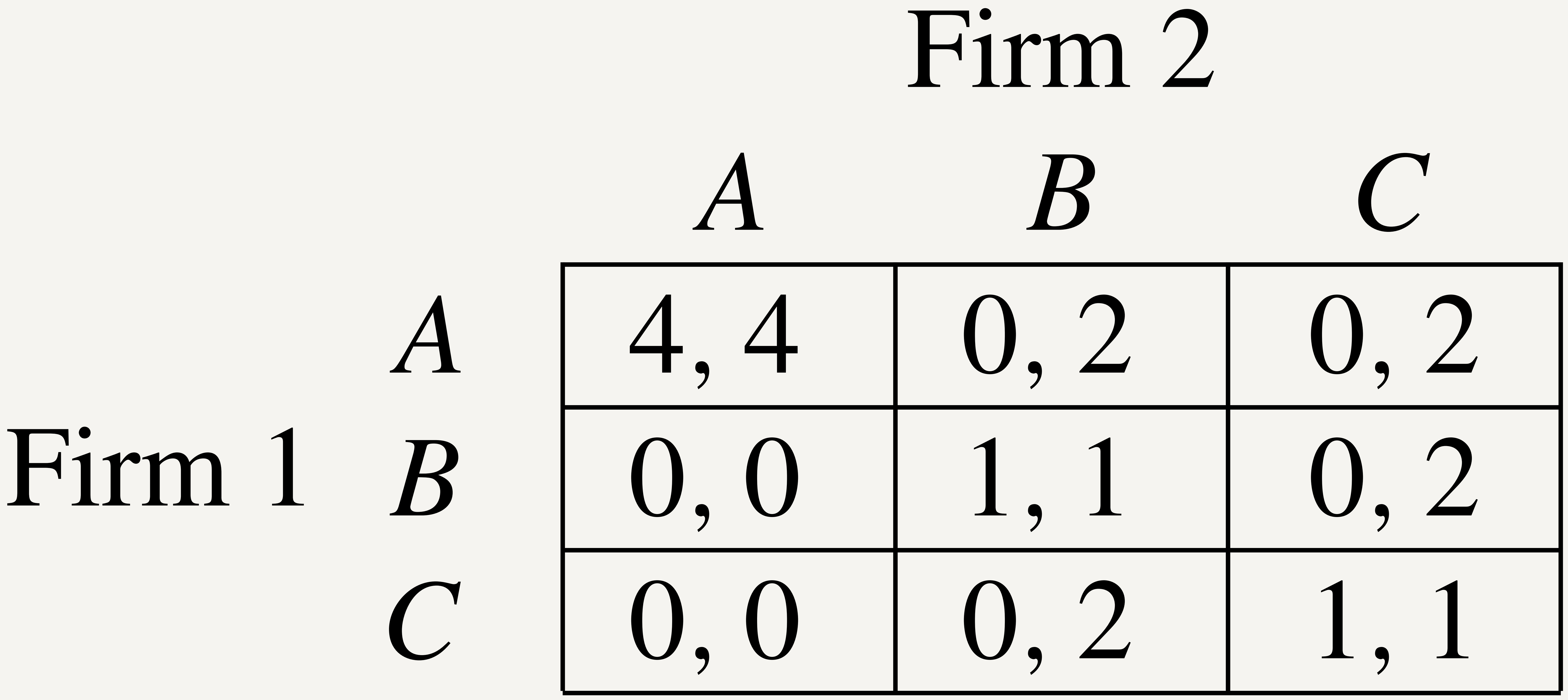
The three-client game



		Firm 2		
		A	*B*	*C*
	A	4, 4	0, 2	0, 2
Firm 1	*B*	0, 0	1, 1	0, 2
	C	0, 0	0, 2	1, 1

Figure 6.6. Three-Client Game

The three-client game



		Firm 2		
		A	*B*	*C*
Firm 1	*A*	4, 4	0, 2	0, 2
	B	0, 0	1, 1	0, 2
	C	0, 0	0, 2	1, 1

Figure 6.6. Three-Client Game

Neither player has a **strictly dominant strategy**.

The main idea of the Nash Equilibrium is: even when there are no dominant strategies, we should expect players to use strategies that are best responses to each other.

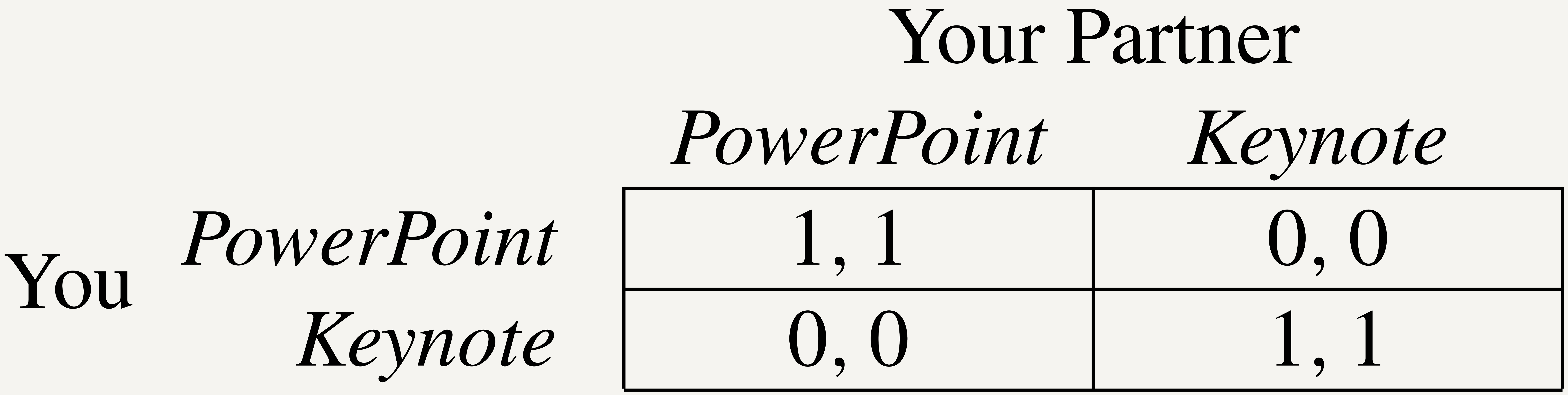
Nash Equilibrium: definition

- Suppose that Player 1 chooses a strategy S and Player 2 chooses a strategy T
- We say that this pair of strategies, **(S, T)**, is a Nash equilibrium if S is a best response to T, and T is a best response to S

Nash Equilibrium: an equilibrium concept

- If the players choose strategies that are best responses to each other, then no player has an incentive to deviate to an alternative strategy
- The system is in an equilibrium state, with no force pushing it toward a different outcome
- The only Nash equilibrium in our example: **(A, A)**

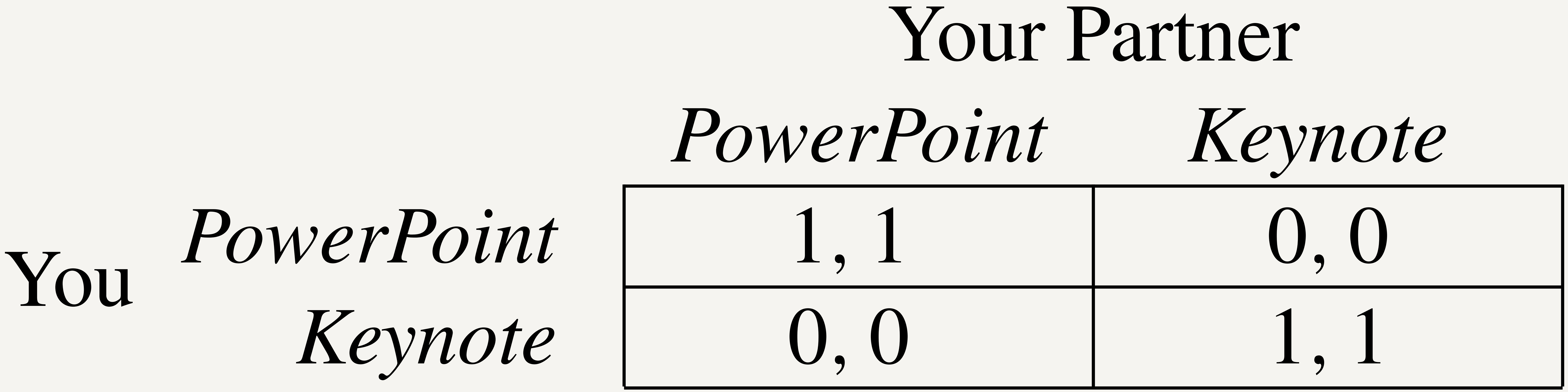
Multiple Equilibria: a coordination game



		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 1	0, 0
	Keynote	0, 0	1, 1

Figure 6.7. Coordination Game

Multiple Equilibria: a coordination game

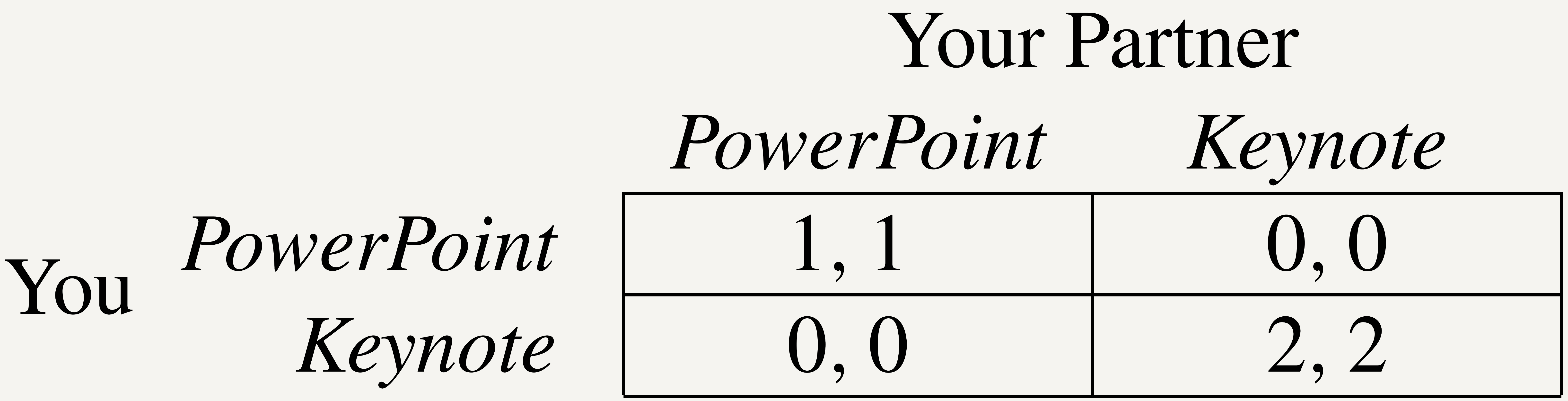


		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 1	0, 0
	Keynote	0, 0	1, 1

Figure 6.7. Coordination Game

Two Nash equilibria: **(PowerPoint, PowerPoint)** and **(Keynote, Keynote)**

An unbalanced coordination game

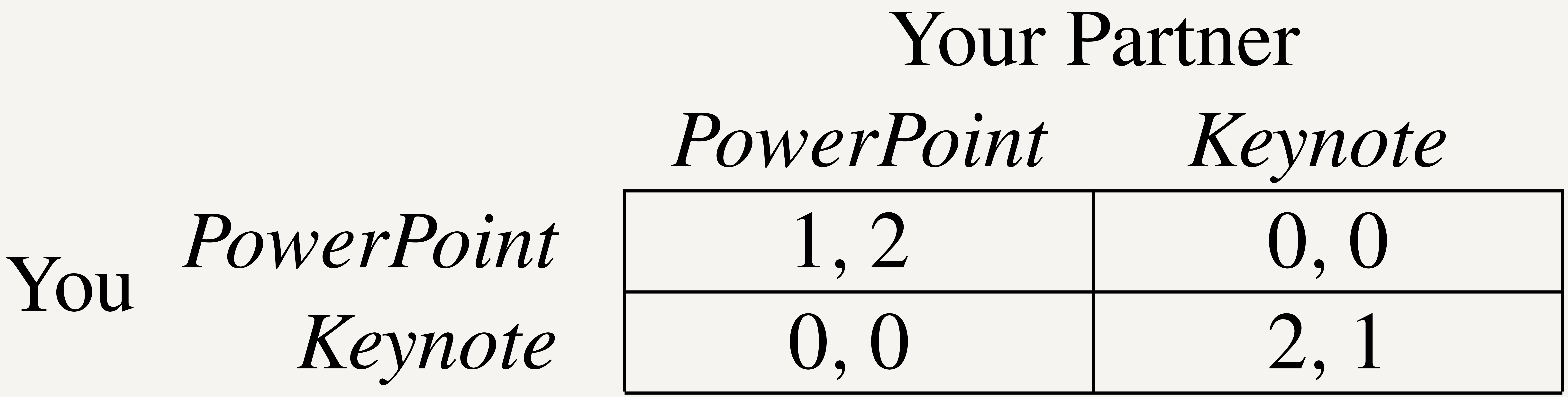


		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 1	0, 0
	Keynote	0, 0	2, 2

Figure 6.8. Unbalanced Coordination Game

- Still two Nash equilibria: **(PowerPoint, PowerPoint)** and **(Keynote, Keynote)**
- But both may choose Keynote, as strategies to reach the equilibrium that gives higher payoffs to both are selected

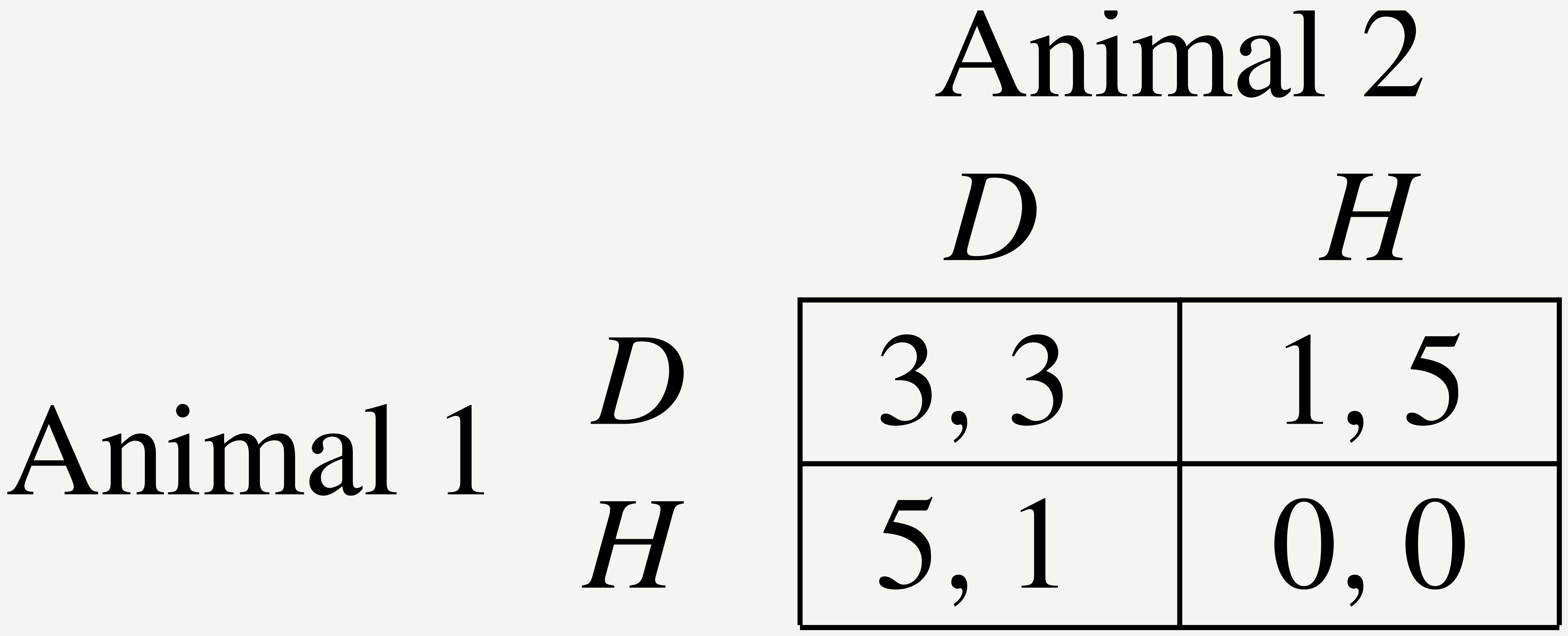
What if you don’t agree with your partner?



		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 2	0, 0
	Keynote	0, 0	2, 1

Figure 6.9. Battle of the Sexes

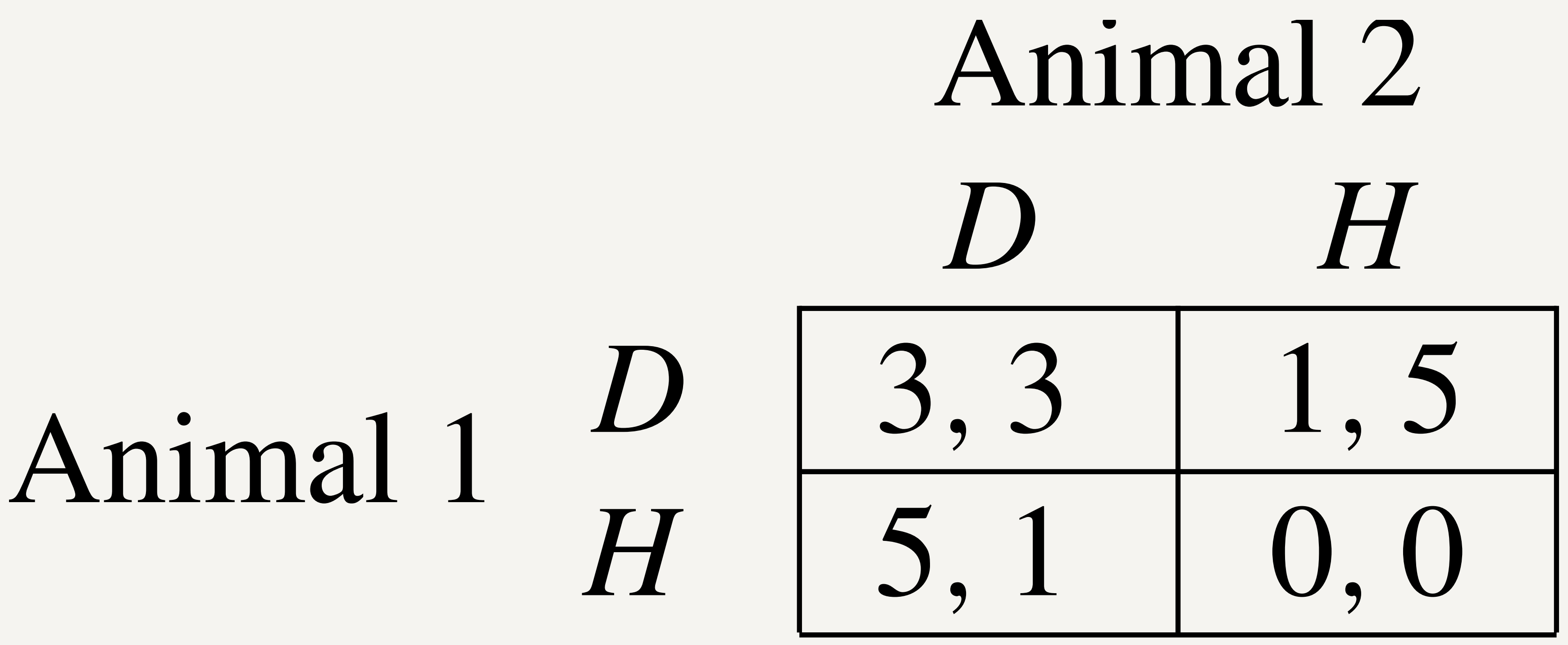
Multiple Equilibria: The Hawk-Dove Game



		Animal 2	
		D	*H*
Animal 1	*D*	3, 3	1, 5
	H	5, 1	0, 0

Figure 6.12. Hawk-Dove Game

Multiple Equilibria: The Hawk-Dove Game

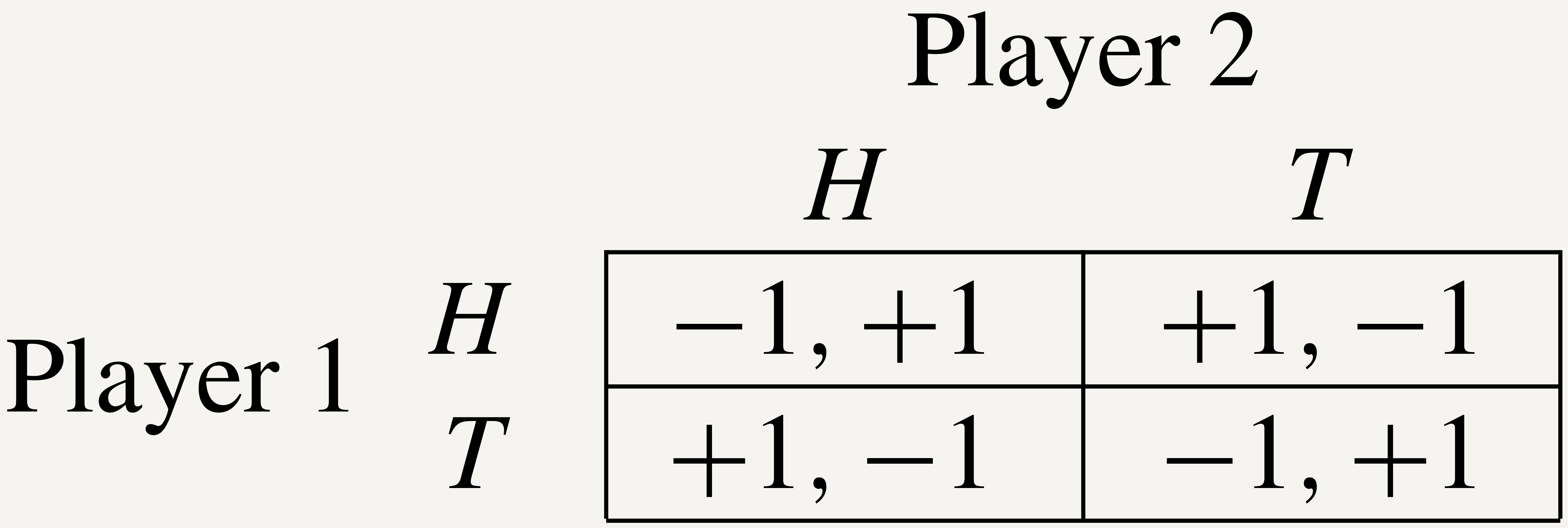


		Animal 2	
		D	*H*
Animal 1	*D*	3, 3	1, 5
	H	5, 1	0, 0

Figure 6.12. Hawk-Dove Game

- Two Nash equilibria: **(D, H)** and **(H, D)**
- The concept of Nash equilibrium helps to narrow down the set of reasonable predictions, but it does not provide a unique prediction!

Matching pennies — a zero-sum game



		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

Figure 6.14. Matching Pennies

- There is no Nash equilibrium for this game, if we treat each player as simply having the two strategies, H or T!
- In real life, players try to make it hard for their opponents to predict what they will play — randomization

Mixed strategies

- Each player chooses a **probability** *p* (*q*) with which he or she will play H (and *1 - p* (*1 - q*) for T)

- We now changed the game to allow a set of strategies corresponding to the interval of numbers between 0 and 1 — **mixed strategies**

  - All previous examples show pure strategies

- But how do we evaluate the payoffs?

The expected value of the payoff

- If Player 1 chooses the pure strategy H while Player 2 chooses a probability of *q* (to play H), as before, then the expected payoff to Player 1 is

$$(-1)(q) + (1)(1 - q) = 1 - 2q$$

- Similarly, if Player 1 chooses the pure strategy T while Player 2 chooses a probability of *q*, then the expected payoff to Player 1 is

$$(1)(q) + (-1)(1 - q) = 2q - 1$$

The expected value of the payoff

- We assume that each player is seeking to **maximize** his expected payoff from the choice of a mixed strategy
- The definition of Nash equilibrium for the mixed strategy version remains the same
  - The pair of strategies is now **(p, q)**

Revisiting the matching pennies game

- No pure strategies can be part of a Nash equilibrium — why?
- What is Player 1's best response to strategy q used by Player 2?
- If $1 - 2q \neq 2q - 1$
  - then one of the pure strategies H or T is in fact the unique best response by Player 1 to a play of q by Player 2
  - because one of (1 - 2q) or (2q - 1) is larger in this case, and so there is no point for Player 1 to put any probability on her weaker pure strategy
  - But we just said pure strategies cannot be part of a Nash equilibrium!
- So we must have $1 - 2q = 2q - 1$
  - **(0.5, 0.5)** is the unique Nash equilibrium for the game

Can a game have both mixed and pure-strategy equilibria?

- You will be indifferent between PowerPoint and Keynote if
$$(1)(q) + (0)(1 - q) = (0)(q) + (2)(1 - q)$$
- Each of you chooses **PowerPoint** with probability 2/3!

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 1	0, 0
	Keynote	0, 0	2, 2

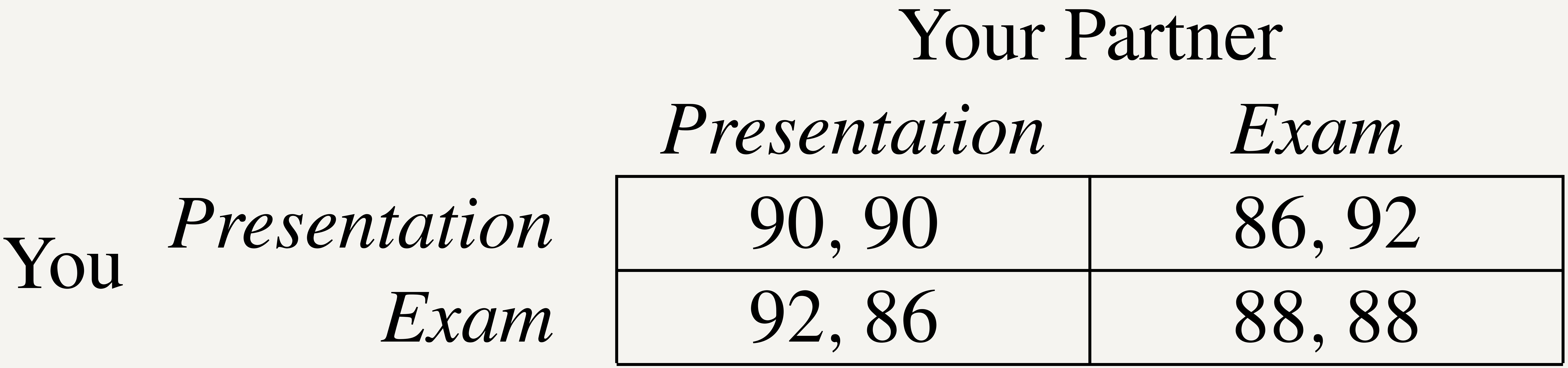
Figure 6.17. Unbalanced coordination game.

What’s good for the society?

- In a Nash equilibrium, each player’s strategy is a best response to the other player’s strategy — they optimize **individually**
    - but we have shown that, as a group, the outcome may not be the best
- We wish to classify outcomes in a game by whether they are “**good for society**”
    - but we need a precise definition of what we mean by this!

A choice of strategies — one by each player — is **Pareto-optimal** if there is no other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff.

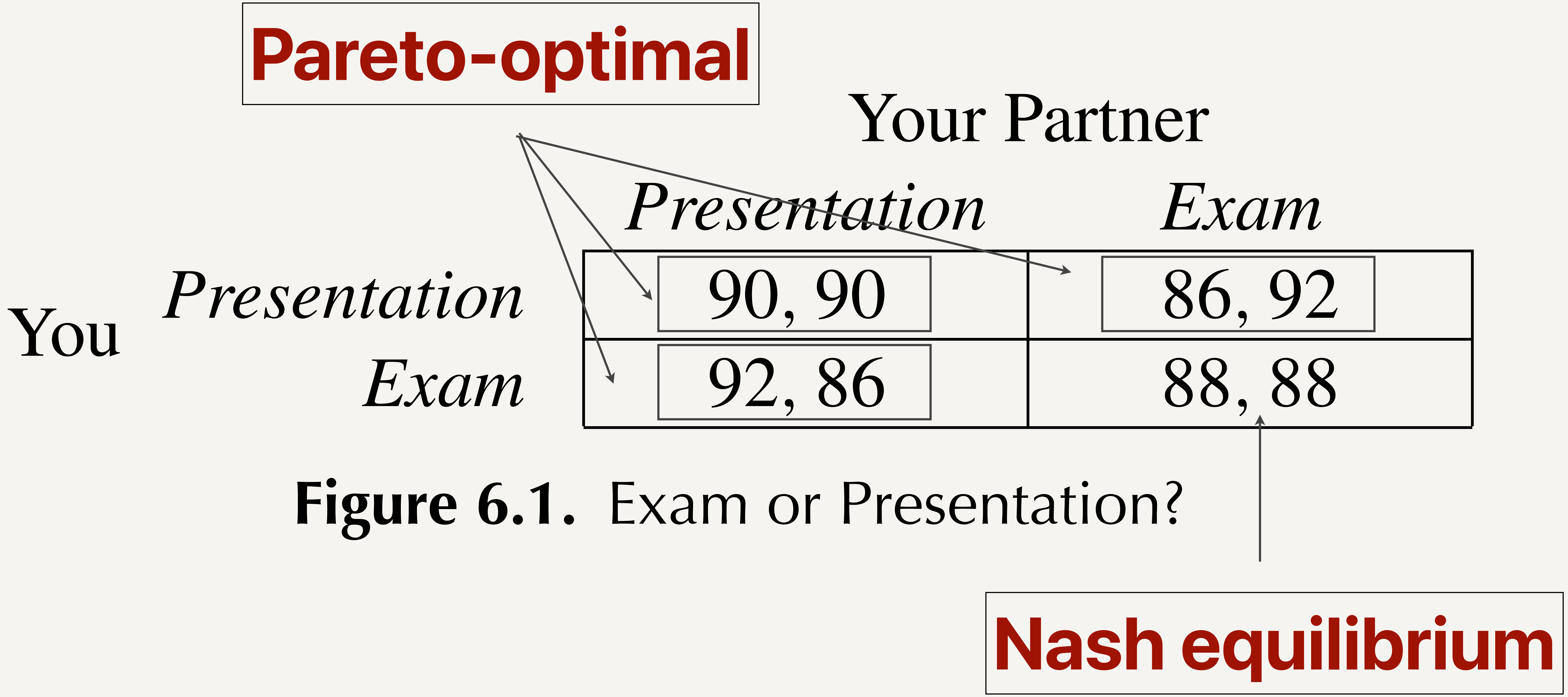
Which choice of strategies is Pareto optimal?



		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1. Exam or Presentation?

Examples of Pareto optimality



Pareto-optimal

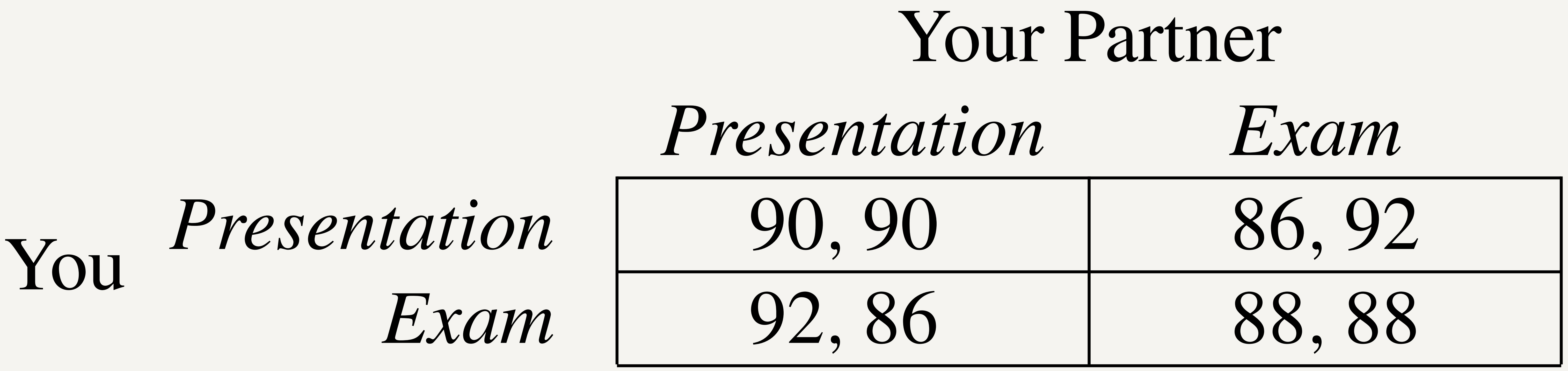
		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1. Exam or Presentation?

Nash equilibrium

Social optimality

A choice of strategies — one by each player — is **socially optimal** if it maximizes the sum of the players' payoffs.



		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1. Exam or Presentation?

Social optimality

If an outcome is **socially optimal**, it must be **Pareto-optimal**, but not the other way around.

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1. Exam or Presentation?

Multiplayer games

- A game with n players, named 1, 2, ..., n, each with a set of possible strategies
  - An outcome (or joint strategy) is a choice of a strategy for each player
  - each player *i* has a payoff function P_i that maps outcomes of the game to a numerical payoff for *i:* for each outcome consisting of strategies $(S_1,S_2,...,S_n)$, there is a payoff $P_i(S_1,S_2,...,S_n)$ to player *i*

Multiplayer games

- A strategy S_i is a best response by Player i to a choice of strategies $(S_1, S_2, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n)$ by all the other players if:

$$P_i(S_1, S_2, \ldots, S_{i-1}, S_i, S_{i+1}, \ldots, S_n) \geq P_i(S_1, S_2, \ldots, S_{i-1}, S_i', S_{i+1}, \ldots, S_n)$$

for all other possible strategies S_i' available to player *i*.

- An outcome consisting of strategies $(S_1, S_2, \ldots, S_n)$ is a Nash equilibrium if each strategy it contains is a best response to all the others

Strictly dominated strategies

- We understand that if a player has a strictly dominant strategy, it will play it — but this is pretty rare!
- Even if a player does not have a dominant strategy, she may still have strategies that are dominated by other strategies
  - A strategy is strictly dominated if there is some other strategy available to the same player that produces a strictly higher payoff in response to every choice of strategies by the other players
  - Strategy S_i for player i is strictly dominated if there is another strategy S_i' for player i such that:

    $$P_i(S_1, S_2, \ldots, S_{i-1}, S_i', S_{i+1}, \ldots, S_n) > P_i(S_1, S_2, \ldots, S_{i-1}, S_i, S_{i+1}, \ldots, S_n)$$

    for all choices of strategies $(S_1, S_2, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n)$ by the other players.

The Facility Location Game: dominated strategies

Two firms are each planning to open a store in one of six towns

		Firm 2		
		B	D	~~F~~
	~~A~~	1, 5	2, 4	3, 3
Firm 1	C	4, 2	3, 3	4, 2
	E	3, 3	2, 4	5, 1

Iterated deletion of strictly dominated strategies

- With A and F eliminated, B and E becomes strictly dominated strategies!

		Firm 2	
		~~B~~	D
Firm 1	C	4, 2	3, 3
	~~E~~	3, 3	2, 4

Nash equilibrium

- The outcome of the game is **(C, D)** — which can be proved to be a Nash equilibrium
- Obtained by going through a process called **iterated deletion of strictly dominated strategies**

Weakly dominated strategies

- A strategy is weakly dominated if there is another strategy available that does at least as well no matter what the other players do, and does strictly better against some joint strategy of the other players
  - Strategy S_i for player i is weakly dominated if there is another strategy S_i' for player i such that:

$$P_i(S_1, S_2, \ldots, S_{i-1}, S_i', S_{i+1}, \ldots, S_n) \geq P_i(S_1, S_2, \ldots, S_{i-1}, S_i, S_{i+1}, \ldots, S_n)$$

for all choices of strategies $(S_1, S_2, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n)$ by the other players, and

$$P_i(S_1, S_2, \ldots, S_{i-1}, S_i', S_{i+1}, \ldots, S_n) > P_i(S_1, S_2, \ldots, S_{i-1}, S_i, S_{i+1}, \ldots, S_n)$$

for at least one choice of strategies $(S_1, S_2, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n)$ by the other players.

Deleting weakly dominated strategies

- Deleting weakly dominated strategies may destroy Nash equilibria!

		Hunter 2	
		~~Hunt Stag~~	Hunt Hare
Hunter 1	~~Hunt Stag~~	3, 3	0, 3
	Hunt Hare	3, 0	3, 3

both outcomes are Nash equilibria!

Required reading: Chapter 6